

11.5 Areas of Circles and Sectors



Before

You found circumferences of circles.

Now

You will find the areas of circles and sectors.

Why

So you can estimate walking distances, as in Ex. 38.

Key Vocabulary

• sector of a circle

In Chapter 1, you used the formula for the area of a circle. This formula is presented below as Theorem 11.9.

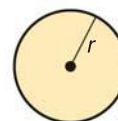
THEOREM

For Your Notebook

THEOREM 11.9 Area of a Circle

The area of a circle is π times the square of the radius.

Justification: Ex. 43, p. 761; Ex. 3, p. 769



$$A = \pi r^2$$

EXAMPLE 1

Use the formula for area of a circle

Find the indicated measure.

a. Area

$$r = 2.5 \text{ cm}$$



b. Diameter

$$A = 113.1 \text{ cm}^2$$



Solution

a. $A = \pi r^2$

$$= \pi \cdot (2.5)^2$$

$$= 6.25\pi$$

$$\approx 19.63$$

Write formula for the area of a circle.

Substitute 2.5 for r .

Simplify.

Use a calculator.

► The area of $\odot A$ is about 19.63 square centimeters.

b. $A = \pi r^2$

$$113.1 = \pi r^2$$

$$\frac{113.1}{\pi} = r^2$$

$$6 \approx r$$

Write formula for the area of a circle.

Substitute 113.1 for A .

Divide each side by π .

Find the positive square root of each side.

► The radius is about 6 inches, so the diameter is about 12 centimeters.

SECTORS A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector APB is bounded by \overline{AP} , \overline{BP} , and \widehat{AB} . Theorem 11.10 gives a method for finding the area of a sector.

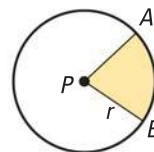
THEOREM

For Your Notebook

THEOREM 11.10 Area of a Sector

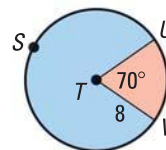
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360° .

$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Area of sector } APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$



EXAMPLE 2 Find areas of sectors

Find the areas of the sectors formed by $\angle UTV$.



Solution

STEP 1 Find the measures of the minor and major arcs.

Because $m\angle UTV = 70^\circ$, $m\widehat{UV} = 70^\circ$ and $m\widehat{USV} = 360^\circ - 70^\circ = 290^\circ$.

STEP 2 Find the areas of the small and large sectors.

$$\text{Area of small sector} = \frac{m\widehat{UV}}{360^\circ} \cdot \pi r^2 \quad \text{Write formula for area of a sector.}$$

$$= \frac{70^\circ}{360^\circ} \cdot \pi \cdot 8^2 \quad \text{Substitute.}$$

$$\approx 39.10 \quad \text{Use a calculator.}$$

$$\text{Area of large sector} = \frac{m\widehat{USV}}{360^\circ} \cdot \pi r^2 \quad \text{Write formula for area of a sector.}$$

$$= \frac{290^\circ}{360^\circ} \cdot \pi \cdot 8^2 \quad \text{Substitute.}$$

$$\approx 161.97 \quad \text{Use a calculator.}$$

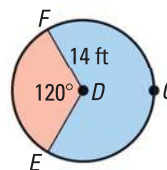
► The areas of the small and large sectors are about 39.10 square units and 161.97 square units, respectively.



GUIDED PRACTICE for Examples 1 and 2

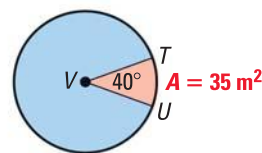
Use the diagram to find the indicated measure.

1. Area of $\odot D$
2. Area of red sector
3. Area of blue sector



EXAMPLE 3 Use the Area of a Sector Theorem

Use the diagram to find the area of $\odot V$.



Solution

$$\text{Area of sector } TVU = \frac{m\widehat{TU}}{360^\circ} \cdot \text{Area of } \odot V$$

Write formula for area of a sector.

$$35 = \frac{40^\circ}{360^\circ} \cdot \text{Area of } \odot V$$

Substitute.

$$315 = \text{Area of } \odot V$$

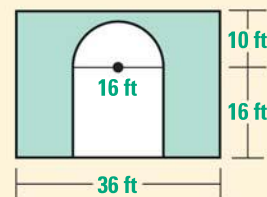
Solve for Area of $\odot V$.

► The area of $\odot V$ is 315 square meters.

**EXAMPLE 4 Standardized Test Practice**

A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?

- (A) 357 ft² (B) 479 ft²
(C) 579 ft² (D) 936 ft²



Solution

AVOID ERRORS

Use the radius (8 ft), not the diameter (16 ft) when you calculate the area of the semicircle.

The area you need to paint is the area of the rectangle minus the area of the entrance. The entrance can be divided into a **semicircle** and a **square**.

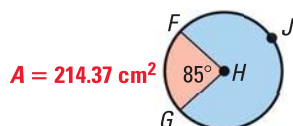
$$\begin{aligned} \text{Area of wall} &= \text{Area of rectangle} - (\text{Area of semicircle} + \text{Area of square}) \\ &= 36(26) - \left[\frac{180^\circ}{360^\circ} \cdot (\pi \cdot 8^2) + 16^2 \right] \\ &= 936 - [32\pi + 256] \\ &\approx 579.47 \end{aligned}$$

The area is about 579 square feet.

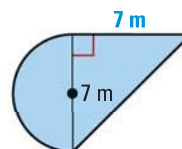
► The correct answer is C. (A) (B) (C) (D)

**GUIDED PRACTICE for Examples 3 and 4**

4. Find the area of $\odot H$.



5. Find the area of the figure.



6. If you know the area and radius of a sector of a circle, can you find the measure of the intercepted arc? *Explain.*

11.5 EXERCISES

HOMWORK KEY

○ = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 7, 17, and 39

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 19, 40, and 42

SKILL PRACTICE

- VOCABULARY** Copy and complete: A ? of a circle is the region bounded by two radii of the circle and their intercepted arc.
- ★ **WRITING** Suppose you double the arc measure of a sector in a given circle. Will the area of the sector also be doubled? *Explain.*

EXAMPLE 1

on p. 755
for Exs. 3–9

FINDING AREA Find the exact area of a circle with the given radius r or diameter d . Then find the area to the nearest hundredth.

- $r = 5$ in.
- $d = 16$ ft
- $d = 23$ cm
- $r = 1.5$ km

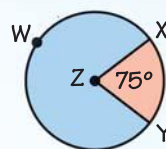
USING AREA In Exercises 7–9, find the indicated measure.

- The area of a circle is 154 square meters. Find the radius.
- The area of a circle is 380 square inches. Find the radius.
- The area of a circle is 676π square centimeters. Find the diameter.

EXAMPLE 2

on p. 756
for Exs. 10–13

- ERROR ANALYSIS** In the diagram at the right, the area of $\odot Z$ is 48 square feet. A student writes a proportion to find the area of sector XZY . *Describe and correct the error in writing the proportion. Then find the area of sector XZY .*



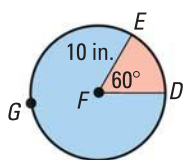
Let n be the area of sector XZY .

$$\frac{n}{360} = \frac{48}{285}$$

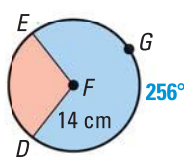


FINDING AREA OF SECTORS Find the areas of the sectors formed by $\angle DFE$.

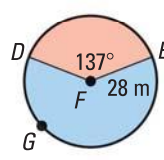
11.



12.



13.

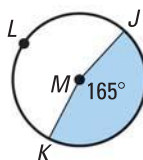


EXAMPLE 3

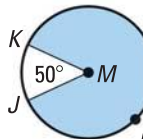
on p. 757
for Exs. 14–16

USING AREA OF A SECTOR Use the diagram to find the indicated measure.

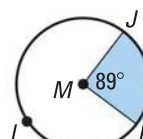
- Find the area of $\odot M$.
- Find the area of $\odot M$.
- Find the radius of $\odot M$.



$$A = 38.51 \text{ m}^2$$



$$A = 56.87 \text{ cm}^2$$



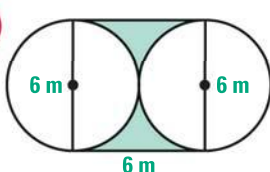
$$A = 12.36 \text{ m}^2$$

EXAMPLE 4

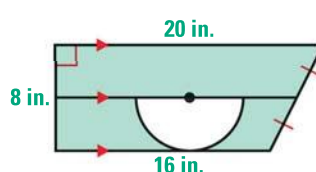
on p. 757
for Exs. 17–19

FINDING AREA Find the area of the shaded region.

17.

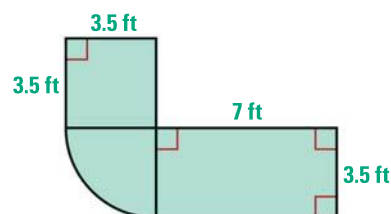


18.



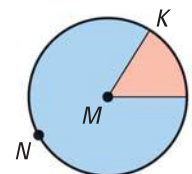
19. ★ **MULTIPLE CHOICE** The diagram shows the shape of a putting green at a miniature golf course. One part of the green is a sector of a circle. To the nearest square foot, what is the area of the putting green?

(A) 46 ft^2 (B) 49 ft^2
(C) 56 ft^2 (D) 75 ft^2

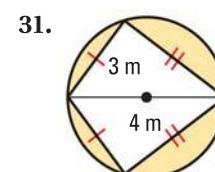
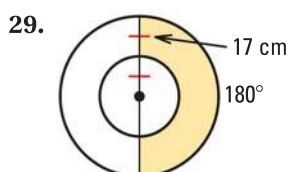
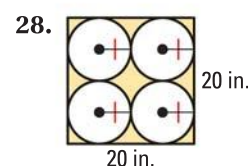
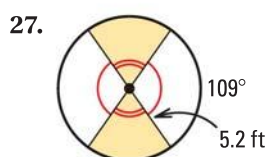
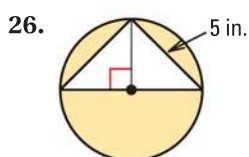


FINDING MEASURES The area of $\odot M$ is 260.67 square inches. The area of sector KML is 42 square inches. Find the indicated measure.

20. Radius of $\odot M$ 21. Circumference of $\odot M$
22. $m\widehat{KL}$ 23. Perimeter of blue region
24. Length of \widehat{KL} 25. Perimeter of red region

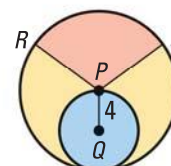


FINDING AREA Find the area of the shaded region.

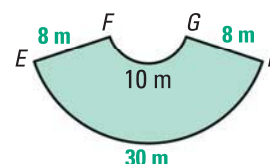


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32. **TANGENT CIRCLES** In the diagram at the right, $\odot Q$ and $\odot P$ are tangent, and P lies on $\odot Q$. The measure of \widehat{RS} is 108° . Find the area of the red region, the area of the blue region, and the area of the yellow region. Leave your answers in terms of π .



33. **SIMILARITY** Look back at the Perimeters of Similar Polygons Theorem on page 374 and the Areas of Similar Polygons Theorem on page 737. How would you rewrite these theorems to apply to circles? *Explain.*
34. **ERROR ANALYSIS** The ratio of the lengths of two arcs in a circle is 2 : 1. A student claims that the ratio of the areas of the sectors bounded by these arcs is 4 : 1, because $\left(\frac{2}{1}\right)^2 = \frac{4}{1}$. *Describe* and correct the error.
35. **DRAWING A DIAGRAM** A square is inscribed in a circle. The same square is also circumscribed about a smaller circle. Draw a diagram. Find the ratio of the area of the large circle to the area of the small circle.
36. **CHALLENGE** In the diagram at the right, \widehat{FG} and \widehat{EH} are arcs of concentric circles, and \overline{EF} and \overline{GH} lie on radii of the larger circle. Find the area of the shaded region.



PROBLEM SOLVING

EXAMPLE 1

on p. 755
for Ex. 37

37. **METEOROLOGY** The *eye of a hurricane* is a relatively calm circular region in the center of the storm. The diameter of the eye is typically about 20 miles. If the eye of a hurricane is 20 miles in diameter, what is the area of the land that is underneath the eye?

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38. **WALKING** The area of a circular pond is about 138,656 square feet. You are going to walk around the entire edge of the pond. About how far will you walk? Give your answer to the nearest foot.

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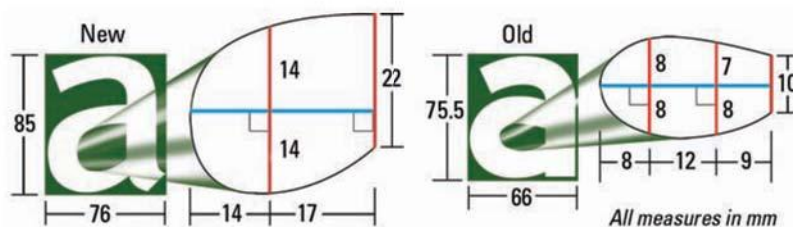
39. **CIRCLE GRAPH** The table shows how students get to school.

- Explain why a circle graph is appropriate for the data.
- You will represent each method by a sector of a circle graph. Find the central angle to use for each sector. Then use a protractor and a compass to construct the graph. Use a radius of 2 inches.
- Find the area of each sector in your graph.

Method	% of Students
Bus	65%
Walk	25%
Other	10%

40. **★ SHORT RESPONSE** It takes about $\frac{1}{4}$ cup of dough to make a tortilla with a 6 inch diameter. How much dough does it take to make a tortilla with a 12 inch diameter? *Explain* your reasoning.

41. **HIGHWAY SIGNS** A new typeface has been designed to make highway signs more readable. One change was to redesign the form of the letters to increase the space inside letters.



- Estimate the interior area for the old and the new "a." Then find the percent increase in interior area.
 - Do you think the change in interior area is just a result of a change in height and width of the letter *a*? *Explain*.
42. **★ EXTENDED RESPONSE** A circular pizza with a 12 inch diameter is enough for you and 2 friends. You want to buy pizza for yourself and 7 friends. A 10 inch diameter pizza with one topping costs \$6.99 and a 14 inch diameter pizza with one topping costs \$12.99. How many 10 inch and 14 inch pizzas should you buy in each situation below? *Explain*.
- You want to spend as little money as possible.
 - You want to have three pizzas, each with a different topping.
 - You want to have as much of the thick outer crust as possible.

43. **JUSTIFYING THEOREM 11.9** You can follow the steps below to justify the formula for the area of a circle with radius r .



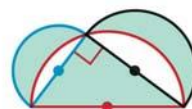
Divide a circle into 16 congruent sectors. Cut out the sectors.



Rearrange the 16 sectors to form a shape resembling a parallelogram.

- Write expressions in terms of r for the approximate height and base of the parallelogram. Then write an expression for its area.
- Explain how your answers to part (a) justify Theorem 11.9.

44. **CHALLENGE** Semicircles with diameters equal to the three sides of a right triangle are drawn, as shown. Prove that the sum of the area of the two shaded crescents equals the area of the triangle.



MIXED REVIEW

PREVIEW

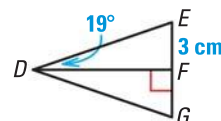
Prepare for
Lesson 11.6 In
Exs. 45–47.

Triangle DEG is isosceles with altitude \overline{DF} . Find the given measurement. *Explain your reasoning.* (p. 319)

45. $m\angle DFG$

46. $m\angle FDG$

47. FG



Sketch the indicated figure. Draw all of its lines of symmetry. (p. 619)

48. Isosceles trapezoid

49. Regular hexagon

Graph $\triangle ABC$. Then find its area. (p. 720)

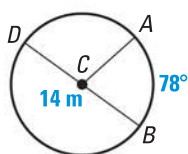
50. $A(2, 2)$, $B(9, 2)$, $C(4, 16)$

51. $A(-8, 3)$, $B(-3, 3)$, $C(-1, -10)$

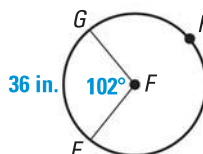
QUIZ for Lessons 11.4–11.5

Find the indicated measure. (p. 746)

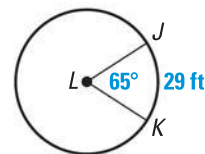
1. Length of \overline{AB}



2. Circumference of $\odot F$

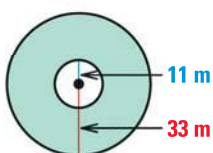


3. Radius of $\odot L$

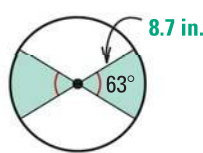


Find the area of the shaded region. (p. 755)

4.



5.



6.

